Business Cycles and Monetary Policy Analysis in a Structural Sticky Price Model of the Lebanese Economy

Jennifer Challita
PHD graduate from the Faculty of Economics and Business Administration, Department of economics, Lebanese University, Branch 1, El Hadath Beirut, Lebanon, jenniferchallita@hotmail.com

Business Cycles and Monetary Policy Analysis in a Structural Sticky Price Model of the Lebanese Economy

Cover Page Footnote

To my family, thank you for encouraging me in all of my pursuits and inspiring me to write my paper. I am especially grateful to my parents, who supported me emotionally.

EDITORIAL

Business Cycles and Monetary Policy Analysis in a Structural Sticky Price Model of the Lebanese Economy

Jennifer Challita

PHD graduate from the Faculty of Economics and Business Administration, Department of Economics, Lebanese University, Branch 1, El Hadath Beirut, Lebanon

Abstract

A medium-size structural model is built in this article in order to define and reshape the Lebanese economy characteristics and features. However, the structural model is featured by a Taylor monetary policy rule design with nominal interest smoothing. Including the Taylor rule helped us to analyze the response function of each variables to different monetary policy shocks (Frish, 1933). Hence, the results of this paper showed how changing coefficients may influence the standard deviations of variables in the model as such the inflation rate, the nominal interest rate and the output GAP (Orphanides and Simon van Norden, 1999). More precisely, the performance of the model is detected while moving the coefficients assuming that monetary authorities' intention is to reduce volatility in the three mentioned variables. Consequently, our findings proved that the coefficient of responsiveness to inflation deviations, the output GAP coefficient and the nominal interest rate coefficient are little bit larger than the ones reported in empirical works for the Euro area[1]. Therefore, these results reflect in some way that the Lebanese Central bank conduct its monetary policy with some responsiveness, in addition, findings ensure that actual behavior of the Lebanese central bank regarding the nominal interest rate setting is well captured by a Taylor rule including the initial proposed coefficients.

Keywords: Structural model, Taylor rule, Inflation rate, Nominal interest rate, Output GAP, DSGE, RBC

1. Introduction

The most investigated subject nowadays, is the real effect of monetary policy on macroeconomic variables. Many analysis undertaken this problematic with different modeling paradigm and originated divergent answers. Structural and reduced form analysis describe two standard paradigms which are broadly used among researchers to examine empirical policies.

The structural modeling is regularly applied in the Real business cycle models (RBC) by means of Dynamic stochastic general equilibrium model. Modeling the state of the macro-economy from micro-economic principles is the main objective of the aforementioned approach. Moreover, structural equations with sticky prices are highly appropriate for business cycles and monetary policies analysis, since they are considered to be independent from any monetary or fiscal policies, also they are competent to describe different short-run effects. However, the approach of the structural models aims to explain the impacts of different monetary policy rules (Barro and Gordon, 1983b) on business cycles properties, they can be also employed for counterfactual and empirical policy analysis, as for the estimation can be done without being subject to the Lucas critique[2].

---

[1] See for example Gerlach and Schnabel (2000) and Clarida, Dali and Gertler (1998).
[2] Following to Muth (1960, 1961) work, Lucas (1977) showed how the reduced-form solutions of forward-looking rational expectation models are functions of the law of motion of the exogenous driving process. For the analysis of monetary policy this means that for any given policy rule the dynamics of the economy, and hence the transmission mechanism, will be different.

Regarding the "vector autoregression" literature, it is a mechanical alternative model which studies monetary policy rules while trying to detect different monetary policy shocks, VAR models are used by many researchers such as Sims and Zha (1995) and Kim (1995), this model proved to be suitable for forecasting purposes and useful in all macro-economics fields. Nevertheless, some limited features of business cycles can be experienced within this kind of reduced form models (VAR models), In other words, their effectiveness for policy analysis is not ultimate for two main reasons. First, from a policy analysis viewpoint, their most deficiency are related to the fact that they are subject to the Lucas critique. Second, the models constructions are not initiated on micro-foundations.

Considering a structural Model for the Lebanese economy, an empirical model defined by both statistical and economic assumptions that rationalize and generate all likely observable results and outcomes. What is the impact of the measured monetary policy within this structural model, on the economic situation in Lebanon? What is the effect of different monetary policy rules on business cycles properties in Lebanon?

In other terms, the research methodology of this thesis is characterized by a process that uses collected data and information in order to make business and economic decisions. The research study presents a structural medium-size model made up of six equations modeling the relationship among monetary and economic variables in Lebanon, with the objective to investigate Lebanese implemented monetary policies and their effects of business cycles patterns. The demand side of the model is derived by optimizing agents' behavior within an IS/LM perception, this structure is widely used in the literature by McCallum and Nelson (1999), Rotemberg and Woodford (1997) and see also Duesenberry (1949).

2. Literature review

Since the Second World War, an important intellectual trend in macroeconomics emerged and has insisted on behavior optimization related to the part of economic decisions makers, this behavior gives attention to both forward-looking and markets clearing.

As per the dynamic stochastic general equilibrium models, economic agents are considered rational (see Muth, 1961), under certainty these agents are able to make choices and decisions based on inter-temporal optimization (see Slutzky, 1937). Moreover, the decisions of agents today highly depend on their anticipations regarding the economy's look in the future. Hence, what occurs in the future depends on the decisions of today, as well as on unknown disturbances that may hit the economy (see Finn and Prescott, 1982). It is difficult to find the equilibrium of these models and it is impossible to be executed algebraically too. Alternatively, computer simulation techniques are used in order to solve DSGE models.

Current macroeconomic research is well dominated by DSGE models. However, they are quite controversial. One of many criticisms is that while DSGE models are rigorous in principle, the used microeconomic foundations of the related equations are imperfect somehow to replicate a heterogeneous economy. Another deeper criticism claims the notion that all agents are optimally with regard to expected events far off in the distant future is idealistic (see Rotemberg and Woodford, 1993, 1996).

Nevertheless, DSGE models are now one of the most used tools by monetary authorities and central banks to understand in a better way the complexity of the economy's response to proposed policy variations (Arthur, 1988).

Marvin Goodfriend and Robert king (1997), the first researchers to name the New Keynesian perspectives as New Neoclassical synthesis (NNS), in that they have combined classical and Keynesian elements, this combination involves the systematic use of rational expectations and inter-temporal optimization as stressed by Robert Lucas. In their research paper Goodfriend and King showed that the inflation evolution in the NNS models depends greatly on current and expected future markups. Also, they verified that the near-zero inflation rate targeting was achievable within the NNS model, and it is more advantageous for monetary exchange.

Clarida, Gali and Gertler (1999), followed as well the NNS approach by adopting the New Keynesian methodology of nominal price rigidities, while emphasizing their analysis on recent methodological advances in macroeconomic modeling. In their research, they used the dynamic general equilibrium model with money and temporary nominal price rigidities[3].

Also, Smets and Wouters (2003) had adopted and developed a dynamic stochastic general equilibrium model with sticky wages and prices for the euro zone using Bayesian estimation techniques. Their model combined a variable capital utilization rate. Their results showed that Bayesian DSGE models complete better than standard VAR. However, a

---

[3] For more info about this paradigm See, e.g, Goodfriend and King (1997), McCallum and Nelson (1997), Walsh (1998), and the references therein.

significant degree of price stickiness in the euro zone is found.

Christiano, Eichenbaum and Evans (2005), as Smets and Wouters (2003) used a DSGE model along with staggered wages and price contracts. They intended to investigate the evidence of output persistence and inflation inertia that may occurs under a mix of fictions. Their model generates after a policy shock, a hump shaped persistent response in output and inertial response in inflation. Also, the money growth rate and the interest rate move persistently in reverse directions after the hit of a monetary policy shock (see also Dubas et al., 2005).

Furthermore, Merz (1995) in her article "Search in the Labor Market and Real Business Cycle" and Andolfatto (1996), "Business Cycles and Labor-Market search". In their studies they embed a search & matching model with exogenous separations into Dynamic Stochastic General equilibrium model and they find that it leads to amplification and propagation relative to the standard RBC model.

3. The structural econometric model, methodology and advantages

Economics and statistics structures are the two main sources of structural models.

The economics structures are defined as deterministic; consequently, the distribution of noisy deterministic data is not reported directly. However, these structures allow the examiner to understand the effect of any economic behavior given the interactions between the set of economic conditions x and outcomes y.

Regarding the statistical structure, they must be added by structural econometric modelers, these modelers can justify why data is not perfectly described by the economic theory. Estimation of alternative assumptions, and examination of structural models' sensitivity, are the key advantage in implementing a structural model. Fundamentally, many reasons are behind specifying and estimating a structural economic model. First, counterfactual situations and policy simulations are well implemented by structural models. Second, there are some unnoticed economic parameters, can be named as behavioral parameters which are only estimated by structural models. Third, the possibility to compare two competing theories in regards to their predictive performance.

4. The model setting

A Sidrauski-type model will be applied in this study, the model abstracts from issues associated to the Lebanese economy. In this model both supply and demand side are structural. Consequently, the developed behavioral equations are derived from individuals' decisions optimization, these behavioral equations highly assume the New-Keynesian hypothesis and paradigm. Accordingly, it accents on real and nominal rigidities, and on the aggregate demand role in determining the national output, along with a stochastic, general equilibrium dynamic real business context.

The following model is an extension of the Keynesian model which incorporates the investment in capital goods and the role of money as medium of exchange. However, it includes essential microeconomic foundations for having main role in the specification of the model. Macroeconomic functions are established as well:

- The demand and supply (production) function
- The budget constraints
- The consumption function
- The investment function
- The money demand function
- The monetary policy rule
- The nominal and real wages

The model is structural because each developed equation has an economic interpretation, any changes in shocks and parameters have equivalents in policy interventions. The impact of policy interventions is examined through the resulting changes in the model outcome.

Since the incorporated main variables are dependent on each other and considered as endogenous. The model is hence described as general equilibrium (Romer, 1994).

To finish, since any random shocks deeply studied by Nelson and Plosser (1982) would highly influence each integrated endogenous variable, the model is called stochastic, and it can be used to measure the uncertainty level in the fundamental model forecasts (see for example Burns and Mitchell, 1946).

5. Model specification and measurement construction

The examined information in the Lebanese market led us to construct six different models. Hence, different functions are represented by these models.

The main agent of the Lebanese economy is represented by a continuum household, this agent has an objective of maximizing both discounted and current future utility values, at time (t). The utility maximization function depends on two main

factors, the consumption preferences shocks ζ, and the level of consumption c.

$$E_t \sum_{j=0}^{\infty} \beta^j U \left(\zeta_{t+j}, c_{t+j} \right) \tag{1}$$

where,

The operator $E_t[.]$ defines the rational expectations, it is conditional to all available information for rational agents at time t, the utility function can be described as follow:

$$U_c > 0, \text{and } U_{cc} > 0$$

$\beta = (1 + \rho)^{-1}$ with $\rho > 0$, is identified as the household's discount factor.

The Household production theory explains that the production of goods is differently produced by each household, also a set of goods is consumed by every household that he purchases from other households. Hence, families are considered both consumers and producers of goods, and c_t represent the aggregate number of set of goods as designated by Stiglitz and Dixit (1977 (Stiglitz, 1977)) where the constant-elasticity indexes are employed.

Other than produced goods, households' purchases are made by using the available transactions technology, thus households need to spend some resources like the output resources according to an approach titled "the output-cost transactions"[4].

A transaction cost will represent the existing transactions technology. The related equation is written as follow:

$$h_t = h(c_t, m_t) \tag{2}$$

Where, h_t denotes the amount of output usages, also called the transactions cost at time (t). This output usages rest on two factors, c_t which represents the number of consumption set and m_t the amount of real balances a household can hold, with$m_t = \frac{M_t}{P_t^A}$, the M_t operator describes the amount of nominal money while P_t^A is the aggregate price level.

The investment equation relation is represented as follow:

$$x_t = k_{t+1} - (1 - \delta) k_t \tag{3}$$

Where x_t represents the invested amount at time (t), in other terms it indicates the increase in capital stock net of depreciation (δ represents the depreciation rate). To add, the amount of x_t is obtained by converting a share of output into capital goods.

Many sources of adjustment costs are expected to show up while capital goods installation. Accordingly, a function $C(x_t)$ will take place, the adjustment costs is provided as output by this function, once x_t units of new capital goods is fitted.

In order to define the amount of output y_t, a production function will be installed. The output amount will be calculated at time t, taking into account the stock of capital k_t, the labor employed n_t and the technology parameter z_t.as well, which is considered as exogenous factor.

$$y_t = f(z_t, n_t, k_t)$$

The households' production function is presumed to occur in a monopolistic competition market, hence referring to Dixit-Stiglitz model, quantities of produced goods are sold in the final goods market. However, the following equation represents the demand function of a single good, where demanded quantity is equal to the production:

$$f(z_t, n_t^d, k_t) = \left(\frac{P_t}{P_t^A} \right)^{-\theta} y_t^A, \text{with } \theta > 1 \tag{4}$$

P_t^A describes the aggregate price level and P_t represents the price market of the household's product. Now, y_t^A denotes the aggregate demand and the relative price constant elasticity is represented by θ.

Two kinds of operations are present within a Labor market: first the household will supply inelastically to this labor market one unit of labor. Second, the labor inputs are purchased by this household at a real wage rate w_t.

Moreover, the number of government bonds (see for example Cebula, 1990) purchased at time t is represented by the variable b_{t+1}. These one-period government bonds have a real price equal to $(1 + r_t)^{-1}$ and each bond is redeemable at $t + 1$, the earned money is used for purchasing one unit of consumption.

The inflation rate in period t, is represented by $\pi_t = \frac{P_t^A}{P_{t-1}^A} - 1$.

As a result, the following represents a household budget constraint:

$$g_t + \left(\frac{P_t}{P_t^A} \right)^{1-\theta} y_t^A - C(x_t) = c_t + x_t + h(c_t, m_t)$$
$$+ w_t(n_t - 1) + m_t - (1 + \pi_t)^{-1} m_{t-1}$$
$$+ (1 + r_t)^{-1} b_{t+1} - b_t \tag{5}$$

---

[4] This model is formally represented by Casares (2000).

r_t denotes the interest rate, and π_t is the rate of inflation, $\pi_t = \frac{P_t^A - P_{t-1}^A}{P_{t-1}^A}$.

Concerning the household real income, two categories can be mentioned: their output production $\left(\left(\frac{P_t}{P_t^A}\right)^{1-\theta} y_t^A\right)$ a specialized output of household η restricted by the demand function, after deducting the adjustments costs of investment $(C(x_t))$, and any government transfers made in favor of them (g_t). This income can be spent as below:

- Consumption (c_t)
- Investment (x_t)
- Payments of the transaction costs $(h(.))$
- Payments of the hired Labor force costs $(w_t(n_t - 1))$, where w_t represents the real wage
- Increasing the holding of real money $(m_t - (1 + \pi_t)^{-1} m_{t-1})$
- Increasing the holding of bonds $((1+r_t)^{-1} b_{t+1} - b_t)$

Households are expected to make rational choices at time t, for the investment, consumption, payments of transaction costs and labor forces, holding bonds and real money and their selling prices. These rational choices are made by maximizing their utility function subject to two functions: the market demand function and the budget constraint (see Thomas and Ali, 1988).

The resulting first order conditions[5] structure (also called Household's optimality conditions) take into account the household's budget constraint and the demand function, in addition to the subsequent functions:

$$U_{ct} - \lambda_t(1 + h_{ct}) = 0 \qquad \left(c_t^{foc}\right)$$

$$-\lambda_t(1 + C_{xt}) + \beta E_t[\lambda_{t+1}(1 - \delta + (1 - \delta)C_{xt+1})] + \beta E_t\left[\xi_{t+1} f_{kt+1}\right] = 0 \qquad \left(K_{t+1}^{foc}\right)$$

$$-\lambda_t(1 + h_{mt}) + \beta\left[E_t\lambda_{t+1}(1 + \pi_{t+1})^{-1}\right] = 0 \qquad \left(m_t^{foc}\right)$$

$$\sum_{j=0}^{\infty}\left((1-\theta)\beta^j\eta^j E_t\left[\lambda_{t+j}\left(\frac{P_t}{P_{t+j}^A}\right)^{-\theta}\frac{y_{t+j}^A}{P_{t+j}^A}\right] + \theta\beta^j\eta^j E_t\left[\xi_{t+j}\left(\frac{P_t}{P_{t+j}^A}\right)^{-\theta-1}\frac{y_{t+j}^A}{P_{t+j}^A}\right]\right) = 0 \qquad \left(P_t^{foc}\right)$$

$$-\lambda_t(1 + r_t)^{-1} + \beta E_t\lambda_{t+1} = 0 \qquad \left(b_{t+1}^{foc}\right)$$

$$-\lambda_t w_t + \xi_t f_{nt} = 0 \qquad \left(n_t^{foc}\right)$$

ξ_t and λ_t symbolize the Lagrange multiplier at time t of the market demand constraint and the budget constraint respectively.

5.1. The consumption function and equation setting

Two main theories determine the real output fluctuations relative to trend, a Neo-classical model and a Keynesian model. The foundation of these models is the mutual interaction between output and spending: income and output are determined by spending, but likewise the spending is determined by income and output.

The consumption function theory was dominated by two main theories since 1950: "The Life-Cycle theory and Permanente-Income Theory of saving and consumption". These contemporary consumption theories emphasize on lifetime decision making (see Campbell and Gregory Mankiw, 1989; Mankiw et al., 1992).

Regarding the equation of consumption first order condition, the Lagrange multiplier described by λ_t, also called the marginal cost of the constraint (consumption shadow price) can be calculated as marginal utility of consumption over one plus the marginal transaction cost.

$$U_{ct} - \lambda_t(1 + h_{ct}) = 0$$

$$U_{ct} = \lambda_t(1 + h_{ct})$$

$$\lambda_t = \frac{U_{ct}}{(1 + h_{ct})}$$

The following is obtained by combining the consumption shadow price and the bonds first order condition (named Monetary policy rule):

$$-\lambda_t(1 + r_t)^{-1} + \beta E_t\lambda_{t+1} = 0$$

$$\lambda_t(1 + r_t)^{-1} = \beta E_t\lambda_{t+1}$$

$$(1 + r_t)^{-1} = \frac{\beta E_t\lambda_{t+1}}{\lambda_t}$$

$$1 + r_t = \beta E_t\left[\frac{(\lambda_{t+1})^{-1}}{(\lambda_t)^{-1}}\right]$$

---

[5] FOC and SOC are used for optimization, they are conditions that determine whether a solution maximizes or minimizes a given function.

$$1 + r_t = \beta E_t \left[\frac{U_{ct}(1 + h_{ct})^{-1}}{U_{ct+1}(1 + h_{ct+1})^{-1}} \right] \tag{6}$$

Since our aim in this study is to reproduce to an acceptable degree the real Lebanese economy, the employed structural model − being part of dynamic stochastic general equilibrium model-must be able to attain its goal. One of the real economy features is a relatively stable growth rate, i.e.: $\frac{\dot{x}}{x} = \gamma$, though $\dot{x}$, a variable with a dot describes the derivative with respect to time.

Consequently, our model needs to entail a constant growth rate at its steady-state (which means it grows at constant exponential rate). Hence a Euler equation will take place:

$$r = \rho - \left(\frac{U''(c).(c)}{U'(c)} \right) \frac{\dot{c}}{c}$$

the household' preferences in this model are characterized by a constant relative risk aversion utility function (CRRA). Our CRRA utility function is presented as follow:

$$U(\zeta_t, c_t) = exp(\zeta_t) \frac{c_t^{1-\sigma}}{1 - \sigma} \tag{7}$$

In addition the transaction technology, or transaction costs function is represented as follow:

$$h(c_t, m_t) = \left\{ \begin{array}{ll} 0 & if \; c_t = 0 \\ b_0 + b_1 \dfrac{c_t^{b_2}}{m_t^{b_3}} & if \; c_t > 0 \end{array} \right\} \tag{8}$$

Now, both functional form (equations (7) and (8)) will be substituted in the first order condition equation (6), then, we will use a technique defined by Uhlig (1999)[6] we get:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \frac{1}{\sigma + (b_2 - 1)h_c^{ss}(.)} (r_t - r^{ss})$$
$$+ b_3 h_c^{ss} \frac{1}{\sigma + (b_2 - 1)h_c^{ss}(.)} (\widehat{m}_t - E_t \widehat{m}_{t+1})$$
$$+ \frac{1}{\sigma + (b_2 - 1)h_c^{ss}(.)} (\zeta_t - E_t \zeta_{t+1})$$
$$\tag{9}$$

$\frac{1}{\sigma + (b_2-1)h_c^{ss}(.)} = \vartheta^c$ represents The semi elasticity[7] of consumption to variations in real interest rate

The equation (9) is rewritten as follow:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \vartheta^c (r_t - r^{ss}) + b_3 h_c^{ss} \vartheta^c (\widehat{m}_t - E_t \widehat{m}_{t+1})$$
$$+ \vartheta^c (\zeta_t - E_t \zeta_{t+1}) \tag{10}$$

A budget constraint including a transaction cost function that delivers a consumption Euler equation, in the latter monetary elements are greatly presented.

5.2. The investment function and equation setting

The capital stock in Lebanon is approximately 3.906 years' GDP, and 17.65 years' worth of investment. Hence, it would take around 20 years' investment flow to rebuild the capital stock to its typical level[8], at usual rates of investment. Accordingly, a high level of investment flows is required for even a small increase in the anticipated level of capital and also a small decrease in the anticipated level of capital shuts the investment down to a dribble. This fact explains why investment is considered an extremely volatile sector of aggregate demand. In addition, it explains why the investment influence on aggregate supply is reduced in the short term. However, over the long run the height of the capital stock in Lebanon is completely determined by the flow of investment and is hence considered a key element of aggregate supply.

The Keynesian theory is described by the circular income flow. This model demonstrates the flow of money within an economy divided into two categories: households and firms.

For classical economists, in case injections and leakages appeared to be in disequilibrium, prices would automatically adjust to restore the equilibrium. While for Keynesian advocates, what would readjust the economy back into equilibrium is not the prices but the level of output.

The capital first order condition K_{t+1}^{foc} **(for the next period)**, will direct the household decision regarding the quantity to invest. As mentioned, the investment equation highly depends on ξ_{t+1} which represents the Lagrange multiplier of the market demand constraint.

As for the labor demand for the next period, the optimality condition (n_{t+1}^{foc}) is attained as follow:

$$-\lambda_t w_t + \xi_t f_{nt} = 0$$

$$\xi_{t+1} = \lambda_{t+1} \frac{w_{t+1}}{f_{n_t+1}} \tag{11}$$

Equation (11) is replaced in the capital first

---

[6] by log-linearizing around the steady state the obtained equation (A first order Taylor expansion).
[7] Semi elasticity measures the percentage change in X to an absolute (not percentage) change in Y.
[8] Author calculation.

order condition of the next period; this will lead to an optimal investment decision:

$$\lambda_t\left(1 + C_{(xt)}\right) = \beta E_t \left[\lambda_{t+1}\left(1 - \delta + (1 - \delta)C_{xt+1}\right. \right.$$
$$\left. \left. + \frac{w_{t+1}}{f_{n_{t+1}}} f_{k_t+1}\right)\right] \tag{12}$$

Once the relation $\beta E_t \lambda_{t+1} = \lambda_t (1 + r_t)^{-1}$ (bonds first order condition) is combined, the Lagrange multiplier wipe out, as a result we get the following equation:

$$1 + C_{xt} = \frac{1 - \delta + (1 - \delta)C_{xt+1} + \frac{w_{t+1}}{f_{n_{t+1}}} f_{k_t+1}}{1 + r_t} \tag{13}$$

At this point, a Cobb−Douglas production function is required with the following form:

$$f(z_t n_t k_t) = exp(z_t) n_t^{1-\alpha} k_t^{\alpha} \tag{14}$$

Yet, an adjustment cost specification is essential, because models with no adjustment costs risk to match cyclical data very poorly:

$$C(x_t) = \varphi x_t^v \tag{15}$$

With $\varphi > 0$ and $v > 1$

Now plugging in the investment first order condition function (13) both functional form, the production function (14) and the adjustment cost function (15), leads to the equation functional form (16), certainly after solving out for $\widehat{x}_t$ and log-linearizing:

$$\widehat{x}_t = (1 - \delta)E_t\widehat{x}_{t+1} + \vartheta^x \psi^{ss} f_k^{ss}\left(E_t\widehat{\psi}_{t+1} + E_t\widehat{f}_{k_{t+1}}\right)$$
$$- \vartheta^x(r_t - r^{ss}) \tag{16}$$

5.3. The money demand function and equation setting

The quantity theory of money is considered as one of the first theories in economics, it dates back to the scholastic writers in the Salamanca school in the mid of 16th century.

Fisher (1911) framed a modern formulation of the quantity theory of money[9] his new version was entitled equation of exchange or cash transaction approach.

Developed by Pigou (1917)[10] and Marshall (1923), the cash balance approach also called the Cambridge approach is a different approach than the

quantity theory of money. The equation of the Cambridge approach[11] is considered more useful than the transaction equation because the value of money is explained in terms of demand and supply for money (see also Nadiri, 1963).

To a certain extent, big similarities exist between the Cambridge approach and the Fisher equation, but the Cambridge approach stress on the idea of "want to hold" rather than "have to hold". Hence, the demand for money depends not only on the transactions volume but also on the level of an individual's wealth along with the opportunity cost of holding money.

The derived functional form of the first order condition m_t^{foc}, define the optimal amount of real balances held by households. By considering both functional form of money demand $-\lambda_t(1 + h_{mt}) + \beta[E_t\lambda_{t+1}(1 + \pi_{t+1})^{-1}] = 0$ and the inter-temporal relation of bonds holding, $\beta E_t\lambda_{t+1} = \lambda_t(1 + r_t)^{-1}$, we get the following equation:

$$E_t\left[(1 + r_t)(1 + \pi_{t+1})\right] = \frac{1}{1 + h_{mt}}$$

Employing the Fisher equation to define the nominal interest rate. Monetary authority by means of nominal interest rate implements the monetary policy R_t:

$$1 + R_t = (1 + r_t)(1 + E_t\pi_{t+1})$$

$$1 + R_t = \frac{1}{1 + h_{mt}} \tag{17}$$

Regarding the h_{mt}, the value was derived from the transaction technology functional form and then inserted in the fisher equation.

Consequently, after log-linearizing we obtained the money demand function:

$$\widehat{m}_t = \frac{b_2}{1 + b_3}\widehat{c}_t - \frac{1}{R^{ss}(1 + b_3)}(R_t - R^{ss}) \tag{18}$$

5.4. The inflation equation: the new Phillips curve, and equation setting

The development of the Phillips curve discipline rose from its successes and failures since 1950, after being a fundamental topic in macroeconomics.

The new Keynesian Phillips curve is a modern version of the Phillips curve; it depends on two

[9] in his book titled "The Purchasing Power of Money".
[10] Pigou, A. C. (1917). The value of money. Quarterly Journal of Economics, 32, (1917−1918), Reprinted in: Readings in Monetary Theory, F. A. Lutz, & L. W. Mints (Eds.), Philadelphia, 1951, pp. 162−183.
[11] Marshall, A. (1923). Money, credit and commerce. London: Macmillan.

microeconomic fundamentals: sticky prices and monopolistic competition between firms. Many competitors exist in the monopolistic competition; these competitors determine the goods prices of their production, worth mentioning that competitors' goods are highly differentiated. Regarding the sticky prices, called nominal rigidities as well, two reasons are behind the price stickiness. First, the cost of price adjustment also called the menu cost. Second, long term fixe contracts, these contracts provide goods at fixed prices.

The first order condition functional form of the selling price (P_t^{foc}), and solving P_t, we get.

$$P_t = \frac{\theta}{\theta - 1} E_t \left[\frac{\sum_{j=0}^{\infty} \eta^j Q_{t,t+j} \frac{w_{t+j}}{f_{n_{t+j}}} \left(P_{t+j}^A\right)^{\theta} y_{t+j}^A}{\sum_{j=0}^{\infty} \eta^j Q_{t,t+j} \left(P_{t+j}^A\right)^{\theta-1} y_{t+j}^A} \right]$$

Proceeding with a log-linearizing of the selling price equation:

$$logP_t = \beta\eta E_t \, logP_{t+1} + (1-\eta)log \, P_T^A + (1-\eta)\widehat{\psi}_t \quad (19)$$

The log-linear percent deviations from steady state:

$$logP_t^A = (1-\eta)log \, P_t + \eta \, log \, P_{t-1}^A \qquad (20)$$

Using the inflation equation, $\pi_t = log \, P_t^A - log \, P_{t-1}^A$ and combining both equations (19) and (20). A new Phillips curve equation is implemented as a result:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\eta)(1-\eta)}{\eta}\psi \qquad (21)$$

5.5. The monetary policy rule and equation setting

The Taylor rule is one of the macroeconomic models that led to a modern way of thinking among policymakers regarding monetary policies (see Woodford, 2001). The rule structures the conduct of monetary policies as an efficient response to incoming data about economic variables as real output and inflation.

The key objective of monetary authorities while conducting monetary policies is the economy stabilization by employing a suitable monetary policy rule.

The Taylor's rule (Taylor 1993) is regularly adopted when the nominal interest rate is the main tool of the monetary policy rule.

$$R_t - R^{ss} = (1 - \mu_3)\left[\mu_1(E_{t-1}\pi_t - \pi^{ss}) + \mu_2 E_{t-1}\tilde{y}_t\right]$$
$$+ \mu_3(R_{t-1} - R^{ss}) + \epsilon_t$$

The $\widehat{y}_t$ describes the current output, it is demand determined as the weighted sum of three variables:

the consumption, the investment that includes the adjustment costs and the transaction costs.

5.6. Nominal and real wages, and equation setting

After 1960, the simple Phillips curve link between unemployment and inflation started to fail because expected or anticipated inflation are not included within. When firms and workers bargain over earnings or wages, they are more concerned about the real value of the wage then the nominal value, hence both parties are more or less willing to amend the level of the nominal wage for any anticipated inflation over the period of the contract.

Two categories of nominal wages contract are regularly signed between two parties. Firstly, "fixed" contract which imply that the nominal wage raises at the inflation' steady state rate. Accordingly, the nominal wage is predetermined at time t as follow:

$$W_t = W_{t-i}(1 + \pi^{ss})^i \cdot$$

As for the second type, the nominal wages are growing at the expected inflation rate, and they are predetermined as follow:

$$W_t = W_{t-1}(1 + E_{t-1}\pi_t) \cdot$$

The average nominal wage is defined by a linear combination of both types:

$$w_t = \kappa\frac{W_{t-1}(1 + E_{t-1}\pi_t)}{P_t^A} + (1-\kappa)\frac{W_{t-1}(1 + \pi^{ss})}{P_t^A}$$

After log-linearizing, the linear real wage equation:

$$\widehat{w}_t = \widehat{w}_{t-1} - \kappa(\pi_t - E_{t-1}\pi_t) - (1-\kappa)(\pi_t - \pi^{ss}) \qquad (22)$$

6. Estimation of the model: data source and results

The model developed by Fagan, Henry and Mestre (2000) is opted as reference for the Lebanese structural modeling. All variables are taken as quarterly observations from quarter 1−1998 till quarter four − 2018. The Real GDP deflated series, inflation rates are extracted from WDI indicator database, and the new reference year for Lebanon is 2010. The quarterly consumption expenditure is taken as a proxy variable for the aggregate demand or consumption function, associated data is also provided from the WDI database. The quarterly time series M1 is employed to measure the monetary aggregate, it includes the currency in circulation + overnight deposits, the time series is derived from Banque du Liban website. More precisely, the M1 is chosen to define the monetary aggregate because it presents the role of money as

medium of exchange, and it is featured by zero nominal rate and perfect liquidity.

The nominal interest rate data is extracted from the Website of Banque du Liban. However, a deflated series is calculated in order to figure out the real interest rate (inflation rates from WDI indicator database ($r_t = R_t - \pi_t$).

6.1. The money demand function estimation

With reference to the engaged model in the empirical part, the Sidrauski-type model, the steady state related to the real interest rate r^{ss} is taken equal to ρ, which is the intertemporal preference. Since the period length in this study is one quarter, then the real interest rate steady state and the intertemporal preference are arbitrarily sat as $\rho = r^{ss} = 0.005$, however this equality suggests a 2% yearly real interest rate. Concerning the inflation steady state baseline it is also equal to 2% per year, hence $\pi^{ss} = 0.005$.

Consequently, the steady state of the nominal interest rate is equivalent to 0.01 since

$R^{ss} = r^{ss} + \pi^{ss} = 0.005 + 0.005 = 0.01$ quarterly, which is correspondent to 4% per year.

The estimation of both consumption and money demand equations helps us to determine the parameters results as σ, b_2, b_3. Regarding the estimation of money demand equation, both coefficients b_2 and b_3 associated to the transaction costs function (equation (8)) are able to determine the consumption elasticities on the one hand, and the nominal interest rate on the other hand from the money demand function.

A dynamic OLS regression[12] is employed in order to estimate b_2 and b_3 by taking the real M1 (m_t), the nominal interest rate (R_t), and the real consumption (c_t).

The standard Johansen Co-integration is the prior test in order to apply the Co-integrating equation estimator. The Johansen Co-integration test calculated by E-views proposes the existence of one long run relationship among the three variables real consumption, M1 and nominal GDP, only for Maximum Eigenvalue test.

6.1.1. Long-run Equilibria: stock Watson Co-integrating regression − DOLS

The real money demand equation: $\widehat{m}_t = \frac{b_2}{1+b_3}\widehat{c}_t - \frac{1}{R^{ss}(1+b_3)}(R_t - R^{ss})$

The approximate long-run relationship between the real Money demand, consumption in constant

prices and Lebanese nominal interest rate is represented by the following equation estimation using the dynamic OLS method and results:

$$m_t = B'X_t + \sum_{j=-k}^{j=L} \overrightarrow{d}_1 \Delta R_{t-j} + \sum_{j=-k}^{j=L} \overrightarrow{d}_2 \Delta c_{t-j} +$$
$$\sum_{j=-k}^{j=L} \overrightarrow{d}_3 \Delta m_t + \zeta_t \tag{23}$$

With X_t matrix of explanatory variables, here the log of Consumption expenditure and the nominal interest rate

$$\log m_t = -35.58 + 2.14 \log c_t - 0.007 R_t$$

$$(0.0000) \quad (0.0000) \quad (0.1339)$$

Following the standard deviations (the number in parenthesis under the estimates), the Lebanese nominal interest rate is not significant to explain changes in the money supply M1. The obtained semi-elasticity of the interest rate implies $\frac{1}{R^{ss}(1+b_3)} = 0.007$, since $R^{ss} = 0.01$, then

$$b_3 = 1,427.57$$

As for the elasticity of the Consumption $\frac{b_2}{1+b_3} = 2.14$, thus $b_2 = 3,057.14$

The Durbin−Watson statistic value is equal to 0.52, a statistic value under two that shows the presence of serial correlations in the residuals. The presence of high serial correlations indicates poor properties of the estimates. Hereafter, it is preferable to compare the test result with other empirical work. Miguel Casares (2001), while studying a money demand equation for the euro zone, confirmed that the monetary aggregate M1 delivers greater stability for this function than M3 data does. Concerning the size of elasticities, it is reported different figures for consumption elasticity and nominal interest rate semi-elasticity, in other words, the size of elasticities in Lebanon are much greater than the euro zone[13]. The consumption is relatively elastic since its coefficient is larger than one. Regarding the nominal interest rate, its negative and small coefficient is inelastic and it shows an inverse relationship between the two variables.

6.2. The consumption function estimation

The parameter b_1 value in the consumption equation characterizes the ratio of steady-state consumption over real money, the said ratio is calculated through quarterly Lebanese data $\left(\frac{c_t}{m_t}\right)$

---

[12] The three time series are stationary in first difference, thus a Dynamic OLS will be ran instead of a static OLS.
[13] In Miguel Casares study (2001) the interest rate semi elasticity is equal to −3.1 while the consumption elasticity is equal to 0.8, a study conducted in the Euro area.

and the produced sample has a mean equal to 1.040 during the period quarter1-1998 till quarter 4−2018.

Regarding the parameter b_0, the constant transaction costs, it was valued in order to suggest that total transaction costs represents 1% of output in steady state $\left(\frac{h^{ss}}{y^{ss}} = 0.01\right)$.

Therefore, the estimation equation is:

$$\frac{h^{ss}}{y^{ss}} = b_0 + b_1 \frac{c_t}{m_t} \tag{24}$$

The estimation results on Eviews system show the following findings:

$$\frac{h^{ss}}{y^{ss}} = 0.2354 + 0.0097 \frac{c_t}{m_t}$$

$$(0.0000) \quad (0.7154)$$

The R-squared value is too low equal to 0.19% while the Durbin−Watson statistic value is equal to 0.1346, which suggests the presence of high autocorrelation in the residual.

Hence $b_1 = 0.2354$

$b_0 = 0.0097$

In the following, the structural equation of the consumption function need to be estimated:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \vartheta^c (r_t - r^{ss}) + b_3 h_c^{ss} \vartheta^c$$
$$(\hat{m}_t - E_t \hat{m}_{t+1}) + \vartheta^c (\zeta_t - E_t \zeta_{t+1})$$

In order to choose the appropriate model for equation estimation, a test for stationary is required. Results of stationarity using three tests PP, ADF and KPSS proved that $r_t - r^{ss}$ and $\hat{m}_t - E_t \hat{m}_{t+1}$ time series are both stationary in level, hence OLS estimation will be employed.

6.2.1. Long-run relationship estimation- OLS

Before proceeding empirically with the model, absent inflation rates and deflation bubbles (variables are deflated or taken in constant prices) helps m_t converges towards $\hat{m}_t$, a steady state value, this aspect remains when consumption and money supply are both not separable in the utility function, as the model presented in this paper.

Estimation results on Eviews generates the following:

$$log\, c_t = E_t\, log\, c_{t+1} - 0.0008 (r_t - r^{ss})$$
$$+ 0.0183 (log\, m_t - E_t\, log\, m_{t+1})$$

$$(0.3405)\ (0.5969)$$

The R-squared value is equal to 11.52% and the Durbin Watson value is equal to 0.65.

Variables such $E_t\, log\, c_{t+1}$ and $E_t \pi_{t+1}$, also called Expectational variables, were replaced by actual observations. The real interest rate is calculated as $r_t = R_t - E_t \pi_{t+1}$. Coeficients' signs are correct and their significance is high.

Referring to these results, the interest rate semi-elasticity is equal to 0.0008, thus $\frac{1}{\sigma + (b_2-1)h_c^{ss}(.)} = 0.0008$

However, the interest rate semi-elasticity is relatively close to σ^{-1} which is the opposite of the degree relative risk aversion coefficient since the marginal consumption cost is minor in steady state $h_c^{ss}(.) = 0.01$[14]. Using the available information, σ in the utility function will be equivalent to 94.44

Regarding the real money balance elements, its theoretical value in the consumption equation is $b_3 h_c^{ss} \frac{1}{\sigma + (b_2-1)h_c^{ss}(.)}$.

To find the steady state result of the model and conclude the corresponding value of $b_3 h_c^{ss} \frac{1}{\sigma + (b_2-1)h_c^{ss}(.)}$, we need to proceed with a calibration of parameters value within the transaction costs function, once done the obtained theoretical value is equal to 11.42

6.3. Cobb−Douglas production function: investment function estimation

Apropos the Cobb−Douglas production function (equation (14)), the related capital share coefficient is set as $\alpha = 0.3$[15]. Hence, this capital share yields to a steady state ratio of consumption over investment equal to 4.8, as consistent to the observations in the Lebanese economy, the sample mean equal is to 4.8 for the ratio c_t/x_t for the period quarter 1−1998 till quarter 4 - 2018

The structural equation (16) is estimated in order to match the variability degree of investment detected in the data. Hence, the semi-elasticity of investment is allocated with respect to the semi-elasticity of the real interest rate ϑ^x. Consequently, an equality is observed between the actual data figures and the variability of investment relative to the variability of consumption.

In order to extract and filter the cyclical components of the both time series investment and Consumption, the Hodrick-Prescott statistical method is adopted. The standard deviation of the transformed series is extracted, we found that the investment

---

[14] In case no marginal costs were considered the interest rate semi elasticity will be equal to σ^{-1} since $h_c^{ss} = 0$

[15] Parameter already estimated for the Lebanese economy in former studies.

standard deviation is about two times and a half the consumption standard deviation. Therefore, the generated ratio of standard deviations is matched through the set of an adequate value for ϑ^x, thus the semi elasticity is equal to $\vartheta^x = \frac{1}{C_x^{ss}(v-1)} = 5$.

Furthermore, the total adjustment cost is represented by 4% of total investment in steady state. Henceforth, both features, the semi-elasticity to the expected return and the size of adjustment costs were used in the adjustment cost function (15) for estimation purpose, and the extractd results are registered subsequent to a study by Miguel Casares (2001) calibration results:

$$v = 2.62$$

$$\varphi = 0.081$$

The depreciation rate is 5% annual, thus 0.0125 per quarter ($\delta = 0.0125$)

6.4. The new Phillips curve estimation

For instance, the probability for a household to maintain a fixed price is the only parameter to estimate in the new Phillips curve, however this probability is equal to 0.75 ($\eta = 0.75$), since in this model it is assumed that household, on average changes his price once a year, this means that the probability to maintain his price is 0.75 and the probability to set a new price is $1 - \eta = 0.25$. Consequently, $(1 - \eta)^{-1} = 4$ is the quarters number without any price change, this number is averagely equal to one year.

6.5. Nominal and real wages estimation

When firms and workers bargain over wages, both parties are concerned with the wage real value, so they are more or less willing to adjust the nominal wage level for any inflation expected within the contract period. Unemployment depends not on the level of inflation, but, rather on the excess of inflation over what was expected. The equation below illustrates this concept:

$$g_w - \pi^e = -\epsilon(u - u^*)$$

Where, g_w represents the rate of wage inflation, π^e denotes the level of expected price inflation, ϵ measures the responsiveness of wages to unemployment (the negative sign signifies that the wages

fall when the unemployment rate exceeds the natural rate, that is when $u > u^*$).

Concerning the nominal wage contracts, they are engaged within a probability κ equal to 0.25, these nominal wages grow at the next period expected inflation. In other terms, every quarter ¼ contracts are revised in order to incorporate expected inflation deviations over target. The remaining contracts entail a nominal increase equal to the inflation steady-state rate.

6.6. Monetary policy rule estimation

It doesn't exist an appropriate calibration for the Lebanese economy in respect to the proposed monetary policy rule by monetary authorities, no previous studies were made for the sample period in this research (1998 quarter 1 till 2018 quarter 4). However, in a previous research paper, we found that the central bank in Lebanon was operating under different instruments and targets. Although the declared direct target of fixing the exchange rate[16], results revealed that monetary policies in Lebanon also respect the Taylor rule while setting the nominal interest rates.

The equation of the structural model has the following form:

$$R_t - R^{ss} = (1 - \mu_3)\left[\mu_1(E_{t-1}\pi_t - \pi^{ss}) + \mu_2 E_{t-1}\tilde{y}_t\right] + \mu_3 (R_{t-1} - R^{ss}) + \epsilon_t$$

In reference to estimation and calibration established on a study conducted by Miguel Casares (2001), the following is fixed as:

$\mu_1 = 1.50$ denotes the inflation rate
$\mu_2 = 0.20$ denotes the output GAP
$\mu_3 = 0.75$ denotes the nominal interest rate

Regarding the stochastic process that hits our system, the shocks are:

A technology shock z_t related to the production function

A consumption preferences shock ζ_t related to the utility function

And finally, a monetary policy rule shock ϵ_t

7. Results analysis for a monetary policy and conclusions

Economists nowadays prefer to suggest simple rules that are dynamic and vigorous to model settings[17]. They suggest that optimal control can be

---

misleading sometimes because it is strictly depending on the definition of the model. Besides, the instrument reaction function that originates implicitly from optimal control rules turns out to be suitable for policy making within a small-size model only. Nevertheless, in large models the high number of included explanatory variables turns the applicability of the monetary policy rules very expensive and subject to many computational errors.

In this study we developed a predictive tool for monetary policy analysis using a structural model described by a Sidrauski-type, a medium-size classical model with monopolistic competition and sticky prices. The model involved multiple structural equations with transactions smoothing money, an inflation equation that represents sticky prices featured by a new Phillips curve, endogenous investment, a real wage equation estiamted from revised and staggered nominal contracts, and a Taylor monetary policy rule with some smoothing components. However, superior and singular treatments were made for structural equations while deriving investment, money demand, consumption and inflation.

The consumption structural equation we estimated is forward looking and it is negatively affected by real interest rates while it is positively impacted by real money balances elements.

As for the investment equation, also assembled as forward looking structural form, it entails 3 explanatory variables with negative signs: the expected marginal product, the expected real marginal cost of capital and the real interest rates. Interestingly, the endogenous investment that resides in the model, allows both market clearing output and capital movements as a result to be endogenous. This outcome is compatible with our findings in previous research, where the Lebanese economy is found better described by an endogenous growth model.

The money demand equation is positively influenced by current consumptions and negatively impacted by nominal interest rates, as the Keynesian policy suggests.

Regarding the supply side, the selling price fixed by a household is forward looking as well, this selling price rests on two variables: the aggregate price level and the marginal real cost, these variables are both positive in the price equation.

The monetary performance implementation of the Lebanese economy is carried out by calculating three standard deviations for the target variables within an estimated model under Taylor's rule μ_1, μ_2, and μ_3. Any variation in these coefficients greatly impacts variability of the output GAP, the inflation rate, and nominal interest rates in order to evaluate the rule stabilization properties. An increase in the coefficient of inflation leads to an increase in both variabilities of the nominal interest rates and the output gap. Any increase in the output gap coefficient leads to a decrease in variability of nominal interest rates and increase in variability of inflation. To close, an increase in nominal interest rates smoothing coefficient does not significantly change neither the variability of inflation, nor the variability of the output gap (Frish, 1933; Orphanides and Simon van Norden, 1999).

To conclude, a large responsiveness to inflation deviations exists, this responsiveness leads to a high output volatility. Though, a moderate degree of interest rates smoothing is recommended for the Lebanese economy as it does not significantly affect either the inflation variability or the output gap, while a drop in the nominal interest rates variability will eventually occur.

Conflict of interest

I have no conflicts of interest to disclose.

References

Arthur, W. B. (1988). Self-reinforcing mechanisms in economics. In P. Anderson, K. Arrow, & D. Pines (Eds.), *The economy as an evolving complex system* (pp. 1313–1337). Redwood City: Addison-Wasley Publishing Company.

Barro, R., & Gordon, D. (1983b). Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics, 12*, 101–121.

Burns, Arthur F., & Mitchell, Wesley C. (1946). *Measuring business cycles New York* (pp. 466–508). New York: National Bureau of Economic Research.

Campbell, John Y., & Gregory Mankiw, N. (1989). *Consumption, income and interest rates: Reinterpreting the time series evidence*. NBER Macroeconomics annual.

Cebula, R. J. (1990). Government borrowing and interest rates in United States: An empirical analysis using IS–LM framework. *Economia Internationale, 43*(2), 159–164.

Dubas, Justin M., Lee, Byung-Joo, & Mark, Nelson C. (2005). Effective exchange rate classifications and growth. In *NBER working papers* (p. 11272). National Bureau of Economic Research, Inc.

Duesenberry, J. S. (1949). *Income, saving and the theory of consumer behavior* (Vol. 9, p. 4). Cambridge: Harvard University Press.

Finn, Kydland, & Prescott, Edward (1982). Time to build and aggregate fluctuations. *Econometrica, 50*, 1345–1370, Nov.

Frisch, Ragnar (1933). *Propagation problems and impulse problems in dynamic economics* (pp. 171–205). Economic Essays in Honour of Gustav Cassel.

Lucas, Robert E. (1977). "Understanding business cycles," carnegie-Rochester conference series on public policy. *Elsevier, 5*(1), 7–29. January.

Mankiw, Gregory N., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *Quarterly Journal of Economics, 107*(2), 407–437 (May 1992).

McCallum, Bennett T., & Nelson, Edward (1999). An optimizing IS-LM specification for monetary policy and business cycle analysis. *Journal of Money, Credit, and Banking, 31*(1), 296–316.

Muth, John F. (1961). Rational expectations and the theory of price movements. *Econometrica, 29*, 315−335.

Nadiri, M. I. (1963). The determinants of real cash balances in the US total manufacturing sector" quarterly. *Journal of Economics, 83*, 173−196. Oliver Jean Blanchard & Peter Diamond, 1990. "The Cyclical Behovior of the Gross Flows of U.S. Workers," Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, vol. 21(2), pages 85-156.

Nelson, Charles R., & Plosser, Charles I. (1982). Trends and random walks in macroeconomic time series: Some evidence and implications. *Journal of Monetary Economics*, 139−162.

Orphanides, Athanasios, & Simon van Norden. (1999). *The reliability of output gap estimates in real time*. Board of Governors of the Federal Reserve System.

Romer, Paul (1994). The origins of endogenous growth. *The Journal of Economic Perspectives*. Winter.

Rotemberg, J., & Woodford, M. (1993). *Dynamic general equilibrium model with imperfectly competitive product markets*. NBER working paper # 4502, October 1993.

Rotemberg, Julio J., & Woodford, Michael (1996). Imperfect competition and the effects of energy price increases on economic activity. Blackwell Publishing *Journal of Money, Credit, and Banking, 28*(4), 550−577. November.

Slutzky, Eugen (1937). The summation of random causes as the source of cyclic processes. *Econometrica, 5*(No. 2), 105−146 (Apr., 1937).

Thomas, Lloyd B., Jr., & Ali, Abderrezak (1988). Anticipated future budget deficits and the term structure of interest rates. *Southern Economic Journal*, 150−161.

Woodford, Michael (2001). *The taylor rule and optimal monetary policy*. Princeton University. unpublished.